Physics Based Animation Fall 2008

# Particle System 1

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# Particle Dynamics

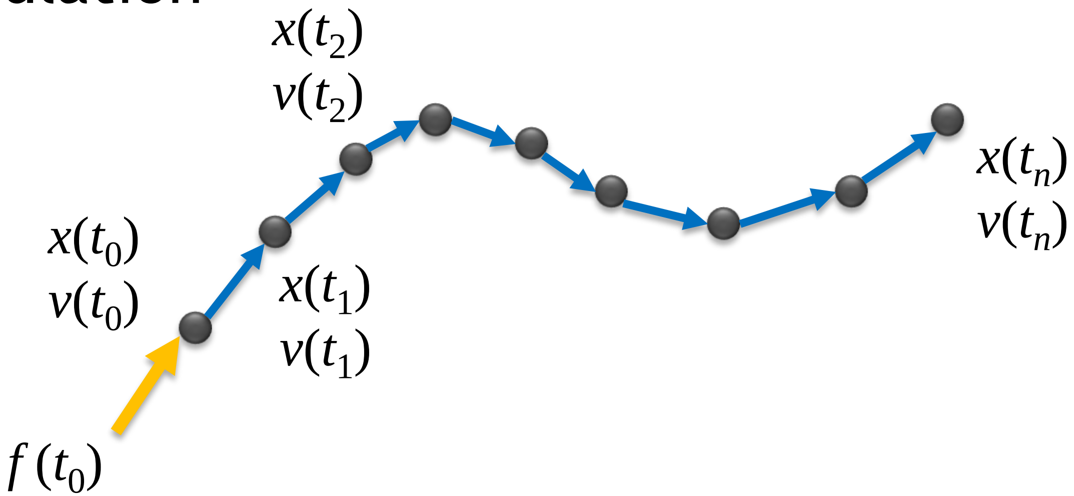
- Idealized body without volume
- Newton's law

$$f = m\ddot{x}$$

- $x$ : position
- $\dot{x} = \frac{dx}{dt}$ : velocity
- $\ddot{x} = \frac{d^2x}{dt^2}$ : acceleration

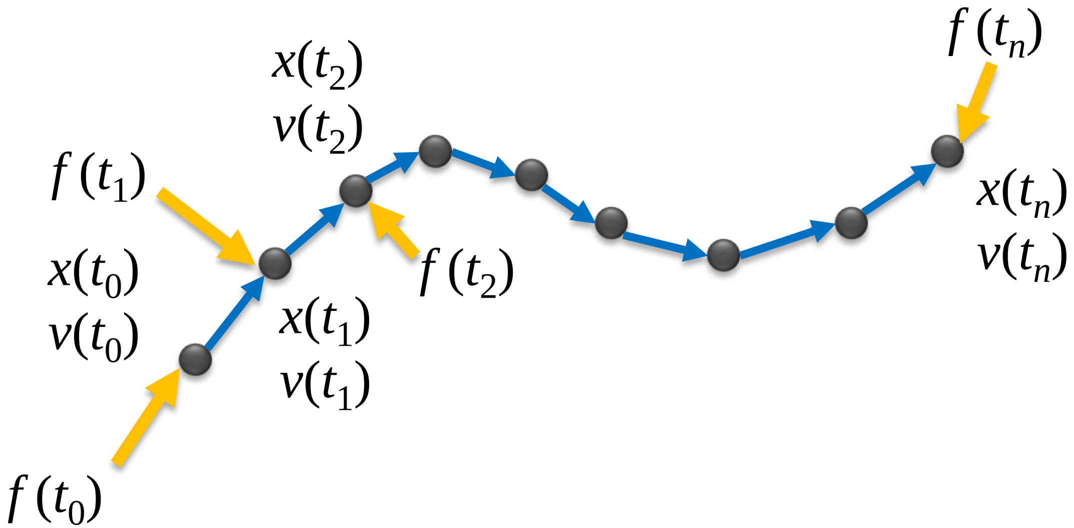
# Typical Problems of Dynamics

- Forward Dynamics
  - Given the force applied to the particle,
    find the acceleration of the particle.
  - In some cases, all the forces are not provided.
    Instead, we have constraint equations.
  - Ex: Simulation

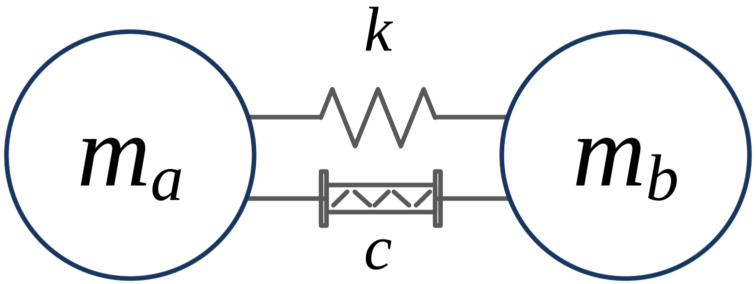


# Typical Problems of Dynamics

- Inverse Dynamics
  - Given the acceleration of the particle, find the force that drives the particle
  - We may have only the trajectory of the particle
  - Ex: Robot motion planning



# Mass-Spring-Damper System

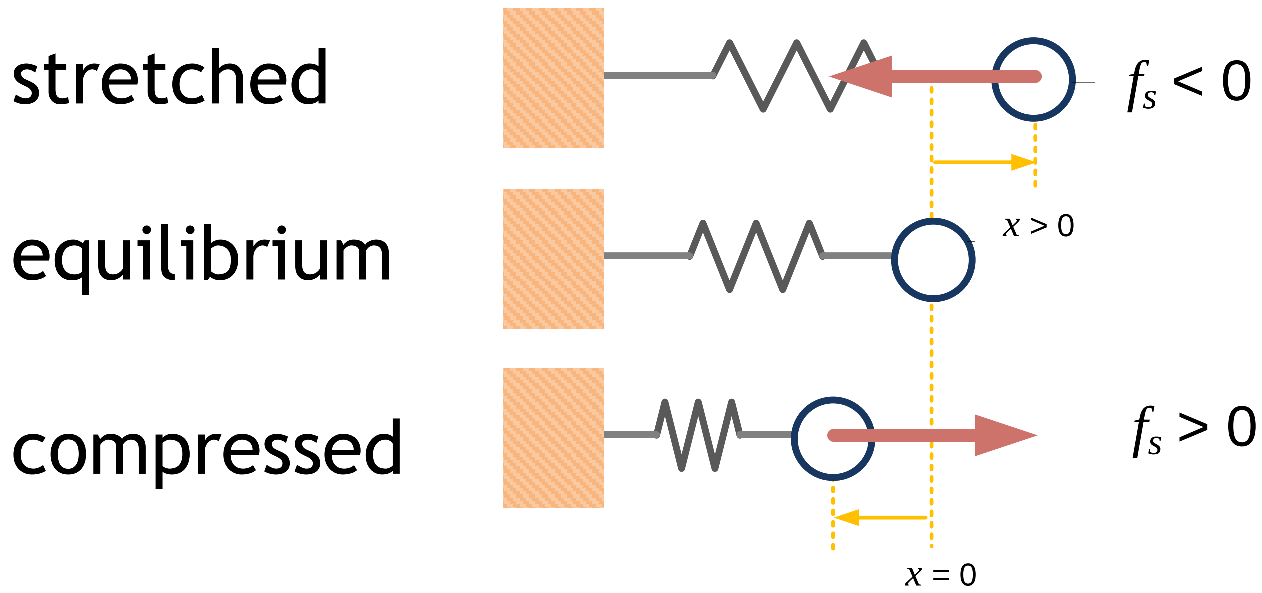



$$f = m\ddot{x}$$

$$f = -kx - c\dot{x}$$

- Easy to implement
- Hard to define proper $k$ and $c$
- Tends to be unstable

# Spring in 1D




$$f_s = -kx$$

# Forward Dynamics

- Evaluate force applied to the particle
  - gravity, spring, damper, collision, …
- Calculate acceleration

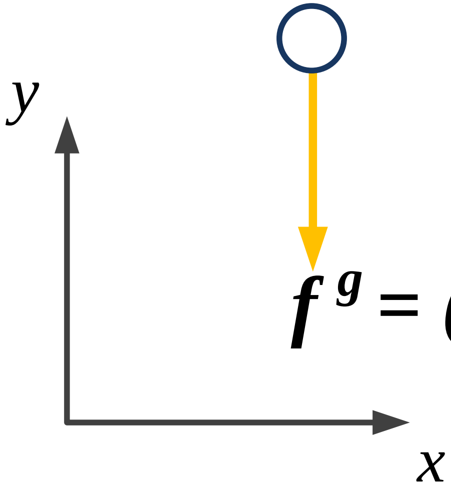
$$a = f / m$$

- Step forward in time
  - Integrate dynamics

$$\ddot{x} = a$$

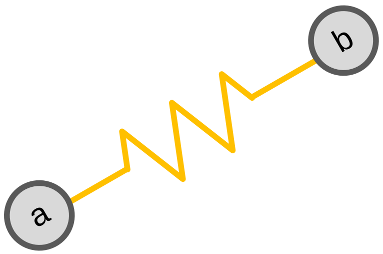
# Gravity Force

- $m$ : mass
- $g$ : gravity, 9.8 m/s$^2$




$$\boldsymbol{f}^{g} = (0, -mg)$$

# Spring Force




- Restoring force
- Resting spring distance = $l_{ab}$
- Magnitude = $k(\|p_b - p_a\| - l_{ab})$
- Direction from body A to body B $d = \dfrac{p_b - p_a}{\|p_b - p_a\|}$

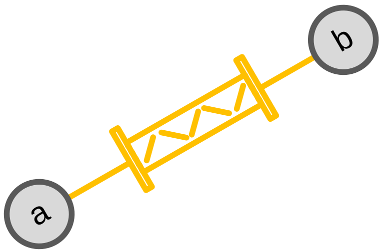
- Force acting on body A

$$f_a^s = \frac{k(\|p_b - p_a\| - l_{ab})}{\|p_b - p_a\|}(p_b - p_a)$$

- Force acting on body B $f_b^s = -f_a^s$

# Damper Force

- Energy consumer due to viscous friction
- Direction from body A to body B




$$d = \frac{p_b - p_a}{\left\| p_b - p_a \right\|}$$

- Velocity of body A on direction $d$

$$v_a^d = \dot{p}_a \cdot d$$

- Force acting on body A

$$f_a^d = c(v_b^d - v_a^d)d = c\big((\dot{p}_b - \dot{p}_a)\cdot d\big)d$$

- Force acting on body B $f_b^d = -f_a^d$

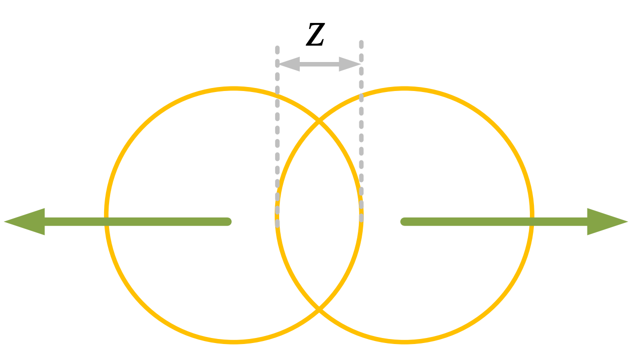
# Collision Force

- Assume a circular shape
- Deeper the penetration, bigger the collision force
  - often called ***penalty method***
- Insert a virtual spring at penetration ( if $z < 0$ )

$$z = \|p_b - p_a\| - (r_a + r_b)$$

$$f_a^c = \frac{k_c z}{\|p_b - p_a\|}(p_b - p_a)$$

$$f_b^c = -f_a^c$$



# Forward Dynamics

- Evaluate force applied to the particle

$$f = f^{g} + f^{s} + f^{d} + f^{c}$$

- Calculate acceleration

$$a = \frac{1}{m} f$$

- Integrate dynamics

$$x(t_{n+1}) = x(t_n) + \Delta t \, v(t_n)$$
$$v(t_{n+1}) = v(t_n) + \Delta t \, a(t_n)$$

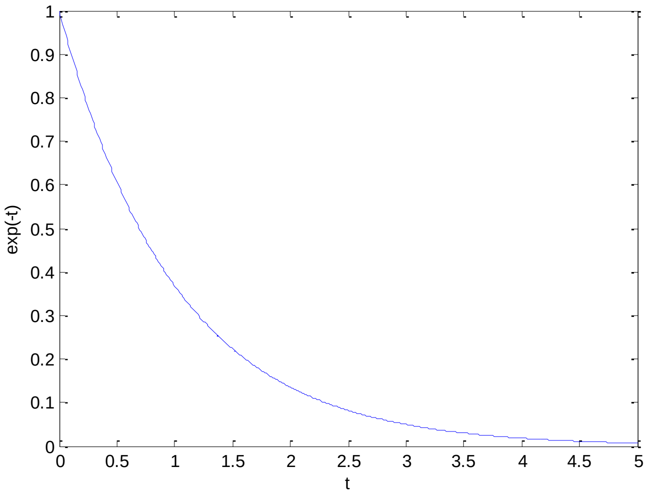
# Numerical Integration of O.D.E

- Consider a simple ordinary differential equation

$$\dot{x} = -kx$$

- We know exact solution

$$x = e^{-kt}$$




- In general, it is almost impossible to find an exac t solution for simple ODEs
  - Instead, we tries to find an ***approximation***

# Taylor Approximation

- Consider a function of t, $f(t)$
- Its Taylor series expansion is

$$f(t) = f(t_0) + \frac{f'(t_0)}{1!}(t - t_0) + \frac{f''(t_0)}{2!}(t - t_0)^2 + \cdots$$

- If $t$-$t_0$ is small,

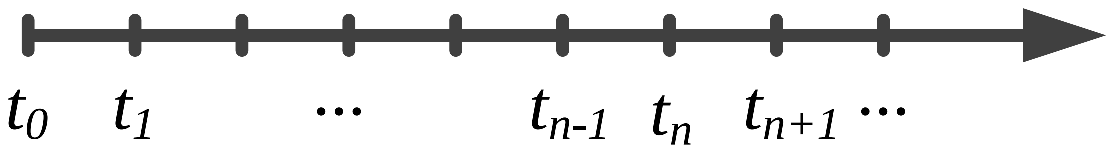
$$f(t) \approx f(t_0) + f'(t_0)(t - t_0)$$

- In previous case,

$$x(t) \approx x(t_0) - kt_0(t - t_0)$$

# Numerical Integration of O.D.E

- Discretize time axis




- Apply Taylor approximation at each time instance
s

$$x(t_{n+1}) = x(t_n) + \Delta t\ \dot{x}(t_n)$$

- $\Delta t = t_{n+1} - t_n$ controls accuracy

# Forward Euler Method

$$x(t_{n+1}) = x(t_n) + \Delta t\, v(t_n)$$
$$v(t_{n+1}) = v(t_n) + \Delta t\, a(t_n)$$

- $1^{st}$ order method
  - error per step is proportional to $\Delta t^2$
  - accumulated error is proportional to $\Delta t$
- Tends to be numerically unstable
  - if $|1\text{-k}\,\Delta t| > 1$, $x(t_n) \rightarrow \infty$ as $n \rightarrow \infty$
  - requires more sophisticated methods for ***accuracy*** and ***stability***